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Analysis of back-office outsourcing contracts for financial services operations

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Managing back-office operations for financial services is a challenging task because of highly volatile and dynamic demand requirements. Lack of service inventories, the inability to backlog demand and significant shortage and overage costs complicate the problem. In such situations, outsourcing all or part of the demand to third-party vendors provides a viable and cost effective option for the firm. Motivated by the remittance processing operations of a Fortune 100 company we examine the usefulness of complementing in-house staffing with different outsourcing arrangements. We study capacity-based and volume-based contracts between a financial services firm and an outsourcing vendor. We examine the impact of demand characteristics on the parameters of contract choice. Through extensive numerical analysis, we ascertain that neither contract is universally preferred, but cost and revenue structures along with demand characteristics determine contract choice.

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1. Introduction

Back-office operations are an integral part of the day-to-day operations of an organization. Such tasks include accounting, human resource management, auditing, information technology and customer-care services. These tasks are the backbone of any organization. In a financial services firm, back-office operations encompass a variety of critical functions involved with the movement of cash and securities and the associated record keeping. These functions come in many forms such as insurance claims processing, mortgage processing, call centres, check processing, credit card application processing and asset tracking to name just a few. Achieving efficiency in these processes is critical to accomplishing overall corporate goals. Creating operationally excellent back offices through streamlined processes is imperative for the smooth running and the future growth of any firm.

One of the most daunting tasks confronting back-office managers is the efficient management of the workforce. Although quite distinct in practice, each of the back-office processes has a common element, the dynamic and stochastic nature of personnel requirements over time. Some of these activities are of a real-time response nature, such as call centres. Others, such as billing and check processing, require a batch-processing response. This paper focuses on the latter category of processes. Even here, there could be a wide range of complexity—for example, billing volumes are more predictable since the firm controls them, than are check-processing volumes, which are dependent on when customers choose to pay their bills. Unlike manufacturing, these jobs are difficult to automate, as most of them require customized service. This makes a large workforce inescapable. However, workforce is an expensive asset and inefficient use of the labour pool can hurt the firm adversely.

Financial transactions are time sensitive—backlogging of demand can lead to delays in posting transactions and significant loss of float. However, at the same time excess capacity leads to idle resource and adds to expense. Our problem is motivated by experience in the credit card remittance processing facility of a Fortune 100 company. The remittance-processing department faces personnel requirements that have considerable demand volatility and uncertainty as seen in Figure 1. Transactions represent revenue to the financial institution that is not realized until they are processed. Hence, the objective is to process the transactions as quickly as possible in order to minimize the loss of float without excessive labour costs.

One option that is widely used in practice to deal with the high variability in demand requirements is outsourcing. Outsourcing provides great opportunities for the firm to take costs out of their operations and increase overall efficiency. There are many compelling reasons for financial services firms to outsource portions of their back-office

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operations, of which cost reduction is an important one. The third party (hereinafter called vendor) to which the financial services provider (hereinafter called *firm*) outsources can provide services at a much lower cost due to economies of scale, demand pooling, specialization and tactical focus. The outsourcing option gives the firm flexibility offered through variable capacity to meet changing business requirements, and allows tighter control of budget by making costs more predictable. According to a research conducted by the financial firm Deloitte, US$356 billion worth of operations would be outsourced by US financial services providers in the 5 years from 2004 to 2009 (Basel Committee on Banking Supervision). This represents 15% of the industry's current cost base. Outsourcing in financial services is a huge global phenomenon that is impossible for managements to ignore. Gartner reports that the global outsourcing market in 2007 was $408 billion and was expected to grow another 8.1% in 2008 to reach $441 billion (Morgan, 2008).

In this paper, we focus on outsourcing contractual agreements for back-office operations between a financial services firm and a third-party vendor under which the firm outsources a portion or the entire process. This mix of in-house and outsourced operations may be necessitated by entirely cost-benefit reasons, or by other operational causes such as the need to do check imaging in-house to enable outsourcing, to retain institutional memory, or for security reasons. Unlike real-time processes such as call centre operations where the firm can decide to not service some of the load based on the service level they decide to maintain (Aksin *et al*, 2008), or outsource calls based on customer segments, batch processes need to serve all of the demand under penalty of shortage or overtime costs. In such a situation, outsourcing imparts much needed flexibility to meet the volatility in demand.

On the basis of our discussions with a manager of an offshore outsourcing vendor we found that financial services providers often outsource all or part of their workload depending on the economic implications of the contract. Here, we examine two versions of contractual outsourcing agreements providing varying levels of flexibility to the firm in meeting the fluctuating demand. In the first contract (OutSFixed), the firm outsources a fixed amount of workload to the vendor. This contract is a form of capacity-reservation in which the firm reserves enough capacity at the vendor's location upfront before realizing demand. In our setting capacity implies the workload that the firm decides to outsource to the vendor. The capacity reserved at the vendor's location may vary for different days of the week (see Figure 2(a)).

In the second contract (InHFixed), the firm keeps a portion of the demand in-house and outsources the variability. This contract can be viewed as a pay-for-work arrangement whereby outsourcing volume is variable. Under this contract the firm pays only for the utilized capacity. Therefore, here the outsourcing volume is determined after demand is realized. As in the OutSFixed contract, here also the amount kept in-house can vary daily (Figure 2(b)). In both the contracts, the vendor has pricing power. We also explore the firm's and the vendor's contract choice problem. To the best of our knowledge in-house processing and outsourcing have not been

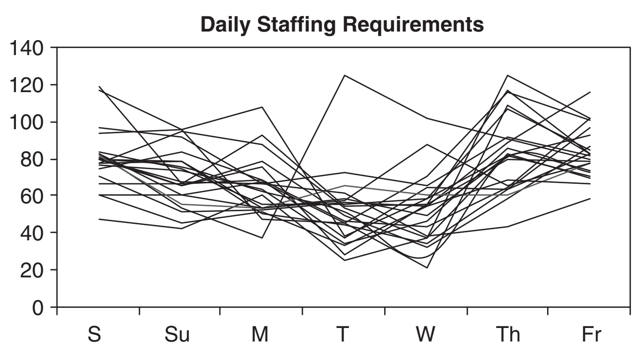

Figure 1 Daily staffing requirements in a credit card remittance facility.
Note: Variations in the Daily Staffing Requirements in a Fortune 100 Company from the data obtained from different weeks. Each line is a different week.

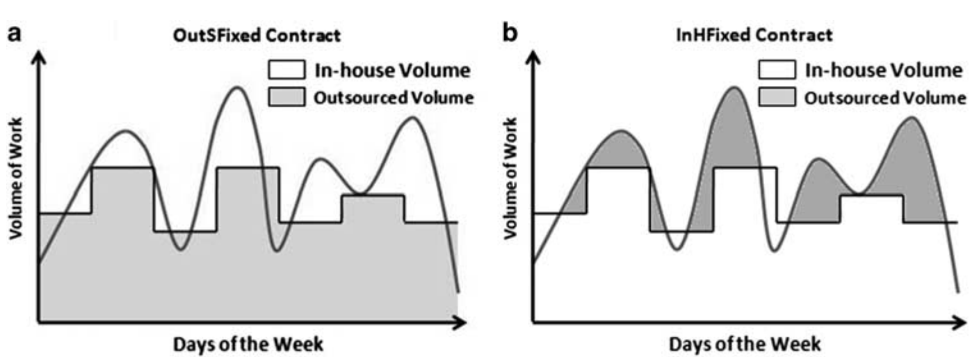

Figure 2 Outsourcing Contracts between the Financial Services Firm and the Vendor. (a) OutSFixed contract; (b) InHFixed contract.

simultaneously considered in the operations management literature in the context of 'batch' type back-office operations under stochastic and dynamic demand.

On first thoughts it might seem that the InHFixed contract may be always beneficial for the firm since under this contract a stable portion of the demand is kept in-house and it is less challenging to deal with stable demand. However, we find that depending on the demand characteristics and the economic parameters of the model, either contract may be preferred by the firm. Our analysis also highlights the conflict of interests of the vendor and the firm in the contract choice, as well as their incentive congruence based on changing demand distributions.

The remainder of the paper is organized as follows. Section 2 provides literature review. Section 3 develops the models for outsourcing contracts. Some structural results are presented in Section 4. The contract choice results appear in Section 5. Managerial insights and future research directions are given in Section 6.

2. Literature review

The issues of outsourcing and in-house employee staffing in financial services operations have been themes of many studies (Moondra, 1976; Mabert, 1979; Larson and Pinker, 2000, Hur *et al*, 2004). The main difficulty in the scheduling of staff in financial services has to do with the variability of demand (Moondra, 1976; Mabert, 1979; Green *et al*, 2007). In practice, to deal with this issue, financial services firms look at outsourcing as an option. However, to the best of our knowledge, literature has not addressed the outsourcing and in-house employee scheduling problem together. Outsourcing in the financial services industry was initially limited to activities that were relatively tangential to the firm's primary business, such as payroll processing. In recent years, however, outsourced activities have included information technology, accounting, audit, investment management and human resources. The most frequently outsourced activity, according to a survey of commercial institutions cited by the Federal Reserve Bank of New York (1999), is some aspect of information technology and human resource functions. In a 2002 survey of North American banks, nearly 75% of bank respondents reported willingness to outsource more functions such as loan documentation, customer servicing and collections (Gursel *et al*, 2002).

Outsourcing contracts in economics and operations management have been extensively studied in the context of manufacturing supply chains (Cachon 2003 and the references therein). This stream of research provides a rich set of models that address supply chain contract design and analysis. The literature on outsourcing contracts for services operations is far more limited and the major emphasis is on call centre outsourcing. For general background on call centre management, staffing and scheduling, we refer the reader to Gans *et al* (2003), Whitt (2006) and Aksin *et al* (2007). Pinedo *et al* (2000) examines call centre operations and various strategic issues related specifically to financial services. Ren and Zho (2008) analyse call centre outsourcing contracts that can coordinate staffing and service quality decisions. Van Mieghem (1999) considers outsourcing contracts where capacity investment of both the call centre and the contractor are decision variables. Aksin *et al* (2008) consider a multi-period model of outsourcing, with multiple decision makers, in the presence of uncertain demand and consider the price as a decision variable for the contractor. Atamtürk and Hochbaum (2001) provide a multi-period treatment of subcontracting, and use call centres as one of their motivating examples. In their paper, demand is deterministic and price is determined exogenously. Nevertheless, extant literature on outsourcing contracts are not concerned with maintaining a mix of in-house and outsourced work as does our paper. Here, we also consider the pricing problem of the outsourcing vendor.

Our focus in this paper is on back-office operations. One key element that differentiates back-office processing in financial services to call centre operations is that they require batch-type processing. Here, *all* work needs to be completed, and there is no question of not handling a portion of the load, an option that is available in real-time operations such as call centre processing depending on the chosen service level. This change of time granularity and requirement to handle all or part of the load manifests itself in modelling such operations differently. We use math programming approach for modelling batch operations (Mabert, 1979; Krajewski *et al*, 1980).

Batch-type operations have been modelled primarily either as generalized set covering problems (Morris and Showalter, 1983; Bailey, 1985) or as goal programming models (eg, Brusco and Johns, 1995; Easton and Rossin, 1996; Azaiez and Al Sharif, 2005). Indeed, goal programming, an approach we use in this paper, has received a great deal of attention among optimization techniques as it attempts to address simultaneously multiple objectives such as maximizing utilization of full-time staff, minimizing understaffing and overstaffing costs, minimizing payroll costs, as well as minimizing deviations from desired staffing requirements, customer special requests, staff preferences and staff special requests. Goal programming models for employee scheduling in batch-type operations can be either deterministic (Goodman, 1974, Baker, 1976; Mabert and Watts, 1982, Chu, 2007) or stochastic (Easton and Rossin, 1996).

Here, we consider multi-period models of outsourcing of back-office processes with multiple decision makers in the presence of demand uncertainty. The financial services firm needs to decide the outsourcing volume and also come up with optimal coverage for in-house processing whereas the vendor decides on the outsourcing price. We analyse two different types of contractual arrangements and study the contract choice problem of each party. The issues of contract choice have been dealt within contract theory by Bajari and Tadelis (2001) and by Aksin *et al* (2008) in the context of construction industry and call centre outsourcing, respectively. In this paper, we combine various features found separately in previous literature on staffing, outsourcing and services subcontracting and incorporate them to develop models that capture the business environment of back-office financial services operations.

3. Model formulation

In modelling back-office operations in financial services, such as remittance processing and claims processing, it is important to note that timely completion of work is of utmost importance. For example, checks that are not processed the same day can result in loss of float. Similarly, delayed claims processing or mortgage application processing can put the firm at a competitive disadvantage. One key element in these operations is the highly stochastic and dynamic nature of demand requirements. As shown in Figure 1, there is considerable amount of variability in the demand requirements for different days of the week. This makes planning for these operations extremely challenging. On the one hand, it is critical to process the jobs as soon as possible to gain float but on the other hand, maintaining excess coverage for these jobs is not profitable as workforce is an expensive asset.

Outsourcing all or some of the demand to a third-party vendor is a cost effective way of dealing with the highly volatile demand requirements. Two of the most widely used contracts in services outsourcing are capacity-based and volume-based contracts. These two contracts can be classified based on whether the firm outsources a stable amount of demand and keeps a variable amount of workload in-house or the other way around where a stable portion of the demand is kept in-house with outsourcing the volatility. In the capacity-based contract, which we call OutSFixed, the firm reserves a certain capacity at the vendor's location to process the jobs. If the realized demand exceeds the reserved capacity then the firm has to cover for the extra demand at its own facility with its in-house employees. In the volume-based contract the firm maintains adequate coverage for a specific level of workload internally and outsources any volume over and above that level to the vendor. We call this type of contract *InHFixed*.

We present models for the above outsourcing contracts that help the financial services firm and the vendor determine the extent of outsourcing and the optimal prices, respectively. On the basis of different cost parameters such as full-time employee costs, costs for under-staffing and over-staffing and outsourcing price quoted by the vendor and the demand characteristics, the firm decides the volume of workload to keep in-house and the amount to outsource. We have modelled the firm's in-house operations explicitly to capture the challenges faced by the firm in managing the demand fluctuations. We have assumed that the vendor has a constant marginal cost for the coverage of unit employee demand outsourced by the firm. The vendor's core competence is in processing the jobs outsourced by the firm and due of greater economies of scale and pooling of resources the vendor can manage the variations or demand fluctuations much more efficiently and at a much lower cost. Therefore, the costs for the vendor in maintaining adequate coverage for outsourced volume is much more stable and the cost of under-staffing and over-staffing is less significant for the vendor than that for the firm. Our models are stochastic in light of the uncertainty associated with the volume as well as volatility of demand in such environments. We formulate a multi-period stochastic goal-programming model to capture the inherent variability in the firm's in-house daily demand requirements. The outsourcing contracts induce a game where the vendor acts as a Stackelberg leader in setting the prices. We assume that demand information and all cost parameters are common knowledge. Even though in real-life settings exact cost information may be difficult to obtain but by exploring the local labour market these personnel costs can be estimated (Aksin *et al*, 2008).

3.1. Fixed volume of outsourced operations contract (OutSFixed)

In this model, for each time period, which can be days of the week, the firm outsources a fixed volume of demand and retains the excess load to be processed in-house. The payment and contract terms in this contract would state that a daily volume V_t^o, would be reserved, and this volume may vary by day of week. If on any day, the realized demand is more than V_t^o, the additional volume is processed in-house otherwise if the realized demand is less than V_t^o then all the demand is outsourced and nothing is kept in-house. Here, the firm needs to pay for the reserved capacity each day whether it utilizes it or not. Under this contract the outsourcing volume is determined before demand realization. The vendor's objective is to determine the optimal price (p^o) to charge for unit employee demand reservation. Here, the vendor acts as a Stackelberg leader in setting the optimal p^o and the firm follows with V_t^o. Next, we define the parameters and the decision variables of the model.

Model parameters:

- K Set of intervals in the discrete probability distribution of staff demand, index $k = 1, \ldots, |K|$
- T Set of time periods, index $t = 1, \ldots, |T|$

E_k Employee demand in interval k of the demand distribution
p_{tk} Probability that employee demand in period t is k
c_f Per day cost of one full-time employee
c_d Per day cost of deficit or over time cost of one employee, or per day cost of a part time/overtime employee
m_d Maximum part time or overtime employees as a percentage of total full-time employees in a day
c_e Per day cost adjustment factor for each overstaffed employee for letting them off early
e^i_{tk} Actual in-house processing volume at outsourced level V^o_t at time t if demand is E_k, $e^i_{tk} = \max(0, E_k - V^o_t)$
c_v Marginal cost incurred by the vendor for the coverage of unit employee demand outsourced by the firm.

Decision variables:
For the firm:

U_t The number of employees scheduled to come in period t
S^+_{tk}, S^-_{tk} The positive and negative deviations from the satisfied employee demand in period t when demand is E_k
V^o_t Outsourced processing volume in period t.

For the vendor:

p^o Vendor price for reserving each unit of employee demand.

The employee demand, E_k, is computed by dividing the volume of transactions that need to be processed by the average processing capability of each employee. For example, if outcome k of demand on a particular day is 300 000 and each employee can handle 3000 transactions on average, then the demand for employees for that outcome that day, E_k, is 100.

The firm's cost minimization model can be stated as follows:

Minimize:

$$\Omega^o = c_f \sum_{t=1}^{T} U_t + c_d \sum_{k=1}^{K} \sum_{t=1}^{T} S^-_{tk} p_{tk} - c_e \sum_{k=1}^{K} \sum_{t=1}^{T} S^+_{tk} p_{tk} + p^o V^o_t$$

subject to

$$\begin{aligned} &U_t + S^-_{tk} - S^+_{tk} = e^i_{tk} && \text{for each } t, k \\ &m_d U_t \geqslant S^-_{tk} && \text{for each } t, k \\ &S^+_{tk}, S^-_{tk} \geqslant 0 && \text{for all } t, k \end{aligned}$$

The objective minimizes the total cost of full-time employees, part-time employees (when there is a deficit), undertime costs and outsourcing costs. The first constraint ensures that the positive (S^+_{tk}) or negative (S^-_{tk}) deviation variables are correctly calculated to capture the deviations between the requirements and the employee coverage on each day for every outcome of in-house volume requirements. The second constraint ensures that the maximum part time/overtime employee requirement is adhered to.

The vendor's profit maximization objective is stated as follows:

Maximize:

$$\pi^o = \sum_{t=1}^{T} (p^o - c_v) V^o_t$$

Before we analyse this contract, we present the second kind of contract, and then analyse them together.

3.2. Fixed volume of in-house operations contract (InHFixed)

In this model, the firm retains a fixed volume of demand to process in-house, and outsources the remaining load to the vendor. The volume retained in-house may vary between days. The payment and contract terms in this contract would state that in-house processing will be done for all daily volume up to V^i_t and any excess demand over this level will be outsourced to the vendor. Under this contract the outsourcing volume is determined after demand realizes. The vendor's objective is to determine the optimal price (p^i) to charge for unit employee demand. As in the OutSFixed model, the vendor here acts as a Stackelberg leader in setting the optimal p^i and the firm follows with the outsourcing volume. The challenge of this model is the correct calculation of outsourced costs while allowing the problem to be modelled as a linear goal programme. Our modelling approach below addresses this issue. The additional notation beyond the OutSFixed Model for this contract is as follows:

Model parameters:

j Possible in-house processing volume levels, $j \in \{0, 1, \ldots, E_K\}$
d^i_{jk} Actual in-house processing volume at level j if demand is E_k, $d^i_{jk} = \min(j, E_k)$
d^o_{jk} Actual outsourced processing volume at level j if demand is E_k, $d^o_{jk} = \max(0, E_k - d^i_{jk})$
C^o_{tj} Total daily outsourcing cost at in-house processing volume level j.

Decision variables:
For the firm:

Y_{tj} An indicator variable to choose in-house processing volume, $Y_{tj} \in \{0, 1\}$
V_t^i In-house processing volume. Derived variable.

For the vendor:

p^i Vendor price for reserving each unit of employee demand.

Firm's cost minimization model can be stated as follows:

Minimize:

$$\Omega^i = c_f \sum_{t=1}^{T} U_t + c_d \sum_{k=1}^{K} \sum_{t=1}^{T} S_{tk}^- p_{tk} - c_e \sum_{k=1}^{K} \sum_{t=1}^{T} S_{tk}^+ p_{tk} + \sum_{j=0}^{E_K} \sum_{t=1}^{T} C_{tj}^o Y_{tj}$$

subject to

$$U_t + S_{tk}^- - S_{tk}^+ = d_{jk}^i Y_{tj} \quad \text{for each } t, k$$

$$m_d U_t \geqslant S_{tk}^- \quad \text{for each } t, k$$

$$C_{tj}^o = 0 \quad \text{for each } t \text{ and } j = 0$$

$$C_{tj}^o = p^i \sum_{t=1}^{T} \sum_{k=1}^{K} d_{jk}^o p_{tk} \quad \text{for each } t \text{ and } j > 0$$

$$V_t^i = \sum_{j=0}^{E_K} j Y_{tj} \quad \text{for each } t$$

$$\sum_{j=0}^{E_K} Y_{tj} = 1 \quad \text{for each } t$$

$$S_{tk}^+, S_{tk}^- \geqslant 0;\ Y_{tj} \in \{0, 1\} \quad \text{for all } t, k, j$$

Note here that the use of d_{jk}^i and d_{jk}^o as preprocessed parameters and the introduction of the Y_{tj} variables enables this contract to be modelled as linear goal programme. An added advantage of this model is that we can accommodate different pricing schemes in the outsourcing contract, for example, if a minimum volume charge, a quantity discount or non-linear pricing structure is agreed with the vendor, this can be easily factored in by suitably changing d_{jk}^o.

Here, the first constraint, as in OutSFixed, ensures that there is adequate employee coverage on each day for every outcome of in-house volume requirements. The second constraint ensures that the maximum part time/overtime employee requirement is met. The third and fourth constraints compute the outsourced cost, and the fifth constraint computes the daily outsourced volume.

Vendor's profit maximization objective is stated as follows:

Maximize:

$$\pi^i = \sum_{t=1}^{T} \sum_{k=1}^{K} (p^i - c_v) d_{jk}^o p_{tk} \qquad \text{for each } j > 0$$

The general equilibrium analysis of these multi-period models in a game theoretic context is extremely difficult. We can, however, determine the optimal prices for each of the above contracts numerically and obtain insights into contract choice. First, we explore some analytical results for the single period model. In Section 5, we study the multi-period problem through numerical examples.

4. Some structural results for both the contracts

In this section, we explore some structural results for the single period versions of both the contracts. We first deduce bounds on the optimal prices charged under each contract, then we study the vendor's profit and firm's cost under increasing demand scenarios.

Lemma 1 *The optimal price charged by the vendor under OutSFixed (p^o) is less than the full-time employee cost of the firm c_f, that is, $p^o < cf$, and when $c_e = 0$, $p^o < c_f$.*

Proof See Appendix. □

The above result gives the upper bound of the prices that the vendor can charge under the OutSFixed contract. By construction, under this contract, the firm absorbs the demand variability with its in-house operations. On the basis of demand forecasts the firm reserves capacity at the vendor's location upfront before demand realizes. The firm meets the variability in demand by giving overtime to its full-time employees. Therefore, the vendor, under no circumstances, can charge a price greater than the firm's full-time employee cost. If the vendor charges a price greater than the firm's full-time employee cost, then the firm prefers to opt for additional full-time employees and outsources zero volume.

Lemma 2 *The optimal price charged by the vendor under InHFixed (p^i) is always greater than or equal to the full-time employee cost of the firm, that is, $c_f \leqslant p^i$.*

Proof See Appendix. □

InHFixed provides greater flexibility to the firm since here the firm can outsource the volatile part of the demand and he has to pay only for the capacity that is utilized. Therefore, if the price charged under InHFixed is less than or equal to the full-time employee cost for the firm it is optimal for the firm to outsource all the volume. Now for any price $\leqslant c_f$, the firm outsources all the volume. Hence, for the vendor it is never optimal to charge any price less than c_f.

Theorem 1 *The optimal price charged under InHFixed is always greater than or equal to the optimal price charged under OutSFixed, that is,* $p^i > p^o$.

Proof The result follows from Lemmas 1 and 2. □

The above result shows that the vendor can charge a higher price under InHFixed than under OutSFixed. Under InHFixed the vendor absorbs the variability in demand and hence the vendor charges a premium for this flexibility. OutSFixed is far more restraining from the firm's point of view because it has to reserve capacity upfront. Owing to this constraining nature of the contract, the vendor cannot charge very high prices under OutSFixed.

Next, we explore the impact of increasing demand requirements on the firm's cost and the vendor's profit functions. We define increasing demand in the following way: for two different vectors of random demand say, X and Y, we say that X is greater than Y if X stochastically dominates Y, that is, $F_X(\cdot) \leqslant F_Y(\cdot)$, where $F(\cdot)$ is the distribution function.

Lemma 3 *Under OutSFixed, at fixed outsourcing price,* p^o, *the outsourcing volume for the firm is non-decreasing in demand.*

Proof See Appendix. □

The above lemma states that under invariant prices for OutSFixed, it is optimal for the firm to meet increments in demand requirements with additional outsourcing volume.

Theorem 2 *The vendor's profit increases with demand under OutSFixed.*

Proof See Appendix. □

As demand volume increases, the firm has to make additional arrangements to cover for the extra volume. This provides added opportunities for the vendor to extract more volume from the firm and in turn increase its revenues. Hence, the vendor's profit increases with demand, the extra volume. This provides added opportunities for the vendor to extract more volume from

Theorem 3 *The vendor's profit and the firm's cost increase with demand under InHFixed if the optimal price charged* p^i *is equal to* c_f, *the full-time employee cost for the firm.*

Proof See Appendix. □

We know from Lemma 2 that at $p^i = c_f$ the firm outsources all the volume under InHFixed. Now under InHFixed the vendor absorbs the variability in demand. So in most of the demand scenarios it is optimal for the vendor to price the contract in such a way that it gets all the volume. If the vendor gets a partial volume from the firm then it is like a double whammy for the vendor. On one the hand, it does not get all the volume and on the other hand the firm keeps a stable portion of demand in-house and in turn outsources a 'more' volatile portion of the demand. So in most cases it is optimal for the vendor to charge a price equal to c_f and get all the volume. If for a certain demand scenario $p^i = c_f$ then the firm outsources all the volume. With increasing demand at $p^i = c_f$ the firm still outsources all the volume and hence the revenues for the vendor increase as the volume outsourced increases.

Analysing the contracts in the multi-period setting is difficult. Note that explicitly modelling the firm's in-house operations renders the models less tractable but at the same time makes them closer to reality. We chose to model the firm's in-house operations explicitly because we wanted to capture the firm's internal processing and outsourcing problems simultaneously. Next through extensive numerical analysis, we explore the contract choice problem for each party.

5. Contract choice results

In this section, we explore the contract choice results. Through extensive numerical analysis we show that based on the economic parameters and the demand characteristics each contract can be beneficial to the firm as well as the vendor. We use discretized β distributions for modelling the demand requirements. The β distribution is ideal for this analysis because of its versatility as it can approximate the shape of diverse probability density functions. By changing the shape parameters, u and v, the β distribution enables modelling of distributions with various shapes and skews. β distribution has been extensively used in a wide variety of applications including staffing (see, eg, Fry *et al*, 2006).

5.1. Contract choice under changing demand volume

Here, we analyse the contract choice results under changing demand volumes. In these examples we took a multi-period setting of 1 week. We considered three values of the firm's full-time cost $c_f = \{\$3, \$6, \$12\}$ which we classify as 'low', 'medium' and 'high' and three levels of demand volumes with means 5, 20 and 35 going from left skewed to right skewed and which are again similarly classified. For the 'low' demand volume we kept the mean of the demand distribution for Monday, Wednesday and Friday at 5 and that for Tuesday and Thursday at 20. For the 'medium'

Table 1 Contract choice under changing demand volume

	Firm's full-time cost					
	Low (c_f = \$3)		*Medium (c_f = \$6)*		*High (c_f = \$12)*	
	OutSFixed	*InHFixed*	*OutSFixed*	*InHFixed*	*OutSFixed*	*InHFixed*
Demand volume						
Low						
OutSVol	280	7	280	15	280	32
Pref	Vendor	Firm	Vendor	Firm	Vendor	Firm
Price	\$1.50	\$12.90	\$3.00	\$12.00	\$6.00	\$12.00
Medium						
OutSVol	98	144	280	144	280	144
Pref		Firm, vendor	Firm	Vendor	Firm	Vendor
Price	\$2.70	\$3.00	\$3.00	\$6.00	\$6.00	\$12.00
High						
OutSVol	280	269	280	269	252	269
Pref	Vendor	Firm	Vendor	Firm	Firm	Vendor
Price	\$3.00	\$3.00	\$6.00	\$6.00	\$11.30	\$12.00

c_d = \$14, $c_e = 0$, $m_d = 50\%$, c_v = \$1.
OutSVol: Volume outsourced by the firm; Price: Price charged by the vendor; Pref: Contract preferred by each party.

demand volume we kept the mean of the demand distribution for Monday, Wednesday and Friday at 20 and that for Tuesday and Thursday at 5. For the 'high' demand volume we kept the mean of the demand distribution for Monday, Wednesday and Friday at 20 and that for Tuesday and Thursday at 35. The support for the β distribution for all the daily demand distributions is kept between 0 and 40. Here, c_d = \$14, $m_d = 50\%$ and c_v = \$1. The excess cost c_e is taken as zero since for most operations it is usually zero (Easton and Rossin, 1996). Table 1 presents the contract choice results.

When the demand volume is low we found that InHFixed is cheaper for the firm and OutSFixed is more profitable for the vendor. At low demands, it is cheaper for the firm to cover for the steady demand with in-house employees and then outsource the volatility rather than reserve capacity upfront. Therefore, even if the vendor charges a high price under InHFixed (\$12.90) it is still cheaper for the firm to go with InHFixed. Under InHFixed the vendor earns revenues only for the utilized capacity. When the demand is low there is very small chance of getting high volumes and hence the vendor does not get substantial volume when the demand is low. Under OutSFixed the vendor charges a low price (\$1.50) and gets all the volume from the firm upfront and earns more profit than what it earns under InHFixed. Next, we focus on the medium demand volumes. At low values of c_f the price charged by the vendor under both the contracts are similar (\$2.70 and \$3.00). As we discussed before, when there is sufficient demand, under InHFixed it is profitable for the vendor to charge a price for which he gets all the volume. The vendor charges a price equal to c_f under InHFixed. At that price the firm outsources all the volume because the price charged is equal to the full-time cost (Theorem 2). Under OutSFixed also the vendor charges a price very close or equal to c_f. Since the price charged is same in both the contracts, InHFixed is cheaper for the firm as it provides more flexibility to him. At low c_f when the demand is medium, it is optimal for the vendor to charge a price (\$2.70) close to c_f under OutSFixed. However, at this price the volume outsourced by the firm under OutSFixed (98) is lower than the volume outsourced under InHFixed (144). Therefore, InHFixed is more profitable for the vendor. At medium and high values of c_f, the price differential between the two contracts is high (\$3.00, \$6.00 and \$6.00, \$12.00), with the vendor charging higher prices under InHFixed. This leads to higher profits for the vendor under InHFixed. However, at this high price InHFixed is more costly for the firm and it prefers OutSFixed.

Finally, we analyse the high demand volumes scenarios. At low and medium values of c_f the vendor charges prices equal to c_f under both the contracts. At similar prices InHFixed is cheaper for the firm because it provides additional flexibility. Now at low and medium values of c_f even if the vendor charges a price equal to c_f under OutSFixed the firm reserves all the capacity upfront (280) and this high outsourcing volume makes OutSFixed more profitable for the vendor. However, at high values of c_f the vendor, under OutSFixed, does not get all the volume from the firm at prices close to c_f (252). Under InHFixed he can still charge a price equal to c_f and get all the volume (269). At high volumes the utilized capacity by the firm is also high and therefore, the vendor also earns more profit under InHFixed.

Table 2 Summary results under changing demand volume

	Firm's full-time cost					
	Low (c_f= \$3)		*Medium* (c_f= \$6)		*High* (c_f= \$12)	
	V(%)	F(%)	V(%)	F(%)	V(%)	F(%)
Demand volume						
Low	46	102	71	177	75	343
Medium			22	2	11	2
High	4	4	4	4	12	3

c_d= \$14, c_e= 0, m_d= 50%, c_v= \$1
V: Maximum loss in profit to the vendor if he agrees to the contract preferred by firm.
F: Maximum increase in cost to the firm if he agrees to the contract preferred by vendor.
Shaded cell depicts the combination where there is incentive congruence.

We find that except for the scenario when the firm's full-time cost is low and demand volume is medium there is no incentive congruence between the vendor and the firm. In all the other cases there is conflict in contract choice for the two parties. Next, we explore the loss in profit for the vendor or the cost increment for the firm if each party has to pick the contract that is preferred by the other under the conflict scenarios. In Table 2, we present these results. We find that at low demand volumes the decrease in profit for the vendor and the increment in cost for the firm is highest in case they pick their least preferred contract. So under these scenarios it is difficult for the negotiation process to result in consensus. However, at medium demand volumes when c_f is at a medium level, the vendor suffers more in his marginal loss of profit whereas the comparative increment in cost for the firm is low. In these cases, the vendor can share some of the additional profit that it earns under his preferred contract and cover the extra costs for the firm and this can lead to win-win situations for both parties. Table 2 identifies the scenarios under which the vendor can come up with revenue-sharing mechanisms that can provide mutually beneficial agreements.

5.2. Contract choice under changing demand volatility

In this section, we explore the contract choice results under changing demand volatilities. We have used coefficient of variation as a measure for demand volatility. To isolate the impact of demand variability we fixed the mean of the β distribution at 20 and varied the coefficient of variation. We considered demand distributions with coefficient variations equal to 0.005, 0.015 and 0.025. We classified the demand distributions as having 'low', 'medium' and 'high' demand volatility. For the 'low' demand volatility we kept the coefficient of variation of the demand distribution for each day of the week at 0.005. For the 'medium' demand volatility we kept the coefficient of variation of the demand distribution for Monday, Wednesday and Friday at 0.005 and that for Tuesday and Thursday at 0.015. For the 'high' demand volatility we kept the coefficient of variation of the demand distribution for Monday at 0.005, Wednesday and Friday at 0.015 and that for Tuesday and Thursday at 0.025. As before the support for the β distribution for all the daily demand distributions is kept between 0 and 40. We consider three values of the firm's full-time cost $c_f = \{\$3, \$6, \$12\}$, which we classify as 'low', 'medium' and 'high', and three levels of demand volatilities. Here $c_d = \$14, c_e = 0, m_d = 50\%$ and $c_v = \$1$. Table 3 presents the contract choice results.

At low demand volatilities OutSFixed is cheaper for the firm and InHFixed is more profitable for the vendor. Here, the price differential between the two contracts is high (\$1.50, \$3.00; \$3.00, \$6.00 and \$6.00, \$12.00), InHFixed being more expensive than OutSFixed. At low demand volatilities when the demand volume is steady, under OutSFixed, the vendor has to charge a price substantially lower than c_f while on the other hand, under InHFixed the vendor gets all the volume even if he charges a price equal to c_f. This high price differential makes OutSFixed cheaper for the firm and also less profitable for the vendor.

Next, we focus on medium demand volatility. At low values of c_f, under both the contracts the vendor charges prices close to c_f (\$2.80 and \$3.00). At similar prices InHFixed is always cheaper for the firm. Now, when c_f is low, the cost differential between c_f and c_d is high as we kept the overtime cost fixed for each of the scenarios. Because of this high cost differential it is expensive for the firm to cover for the variable portion of the demand in-house with overtime. Hence, under OutSFixed, even at prices close to c_f the firm reserves a very high capacity upfront (178). Under InHFixed the firm only pays for the utilized capacity and so even though the firm outsources all the volume the realized volume (144) is lower than the reserved capacity under OutSFixed leading to comparatively smaller profits under InHFixed. At medium values of c_f there is moderate price differential between the two contracts (\$3.70 and \$6.00). Even at this moderate price differential, InHFixed is cheaper for the firm as it offers more flexibility. Here the vendor cannot charge a price close to c_f under OutSFixed. At prices close to c_f the firm prefers to manage the volatility with overtime rather than reserving a high capacity upfront because here c_f is at a medium level and so the cost differential between c_f and c_d is also lower. Under InHFixed the firm charges price equal to c_f and gets all the volume. Because here the vendor charges a higher price (\$6.00) and at the same time gets all the volume the profits under InHFixed is also higher.

Table 3 Contract choice under changing demand volatility

	Firm's full-time cost					
	Low (c_f = \$3)		*Medium (c_f = \$6)*		*High (c_f = \$12)*	
	OutSFixed	*InHFixed*	*OutSFixed*	*InHFixed*	*OutSFixed*	*InHFixed*
Demand volatility						
Low						
OutSVol	280	144	280	144	280	144
Pref	Firm	Vendor	Firm	Vendor	Firm	Vendor
Price	\$1.50	\$3.00	\$3.00	\$6.00	\$6.00	\$12.00
Medium						
OutSVol	178	144	238	144	280	144
Pref	Vendor	Firm		Firm, Vendor	Firm	Vendor
Price	\$2.80	\$3.00	\$3.70	\$6.00	\$6.10	\$12.00
High						
OutSVol	280	132	280	128	280	132
Pref	Vendor	Firm	Vendor	Firm	Firm	Vendor
Price	\$3.00	\$4.80	\$5.40	\$9.00	\$8.20	\$16.30

c_d = \$14, c_e = 0, m_d = 50%, c_v = \$1.
OutSVol: Volume outsourced by the firm; Price: Price charged by the vendor; Pref: Contract preferred by each party.

At high values of c_f, the price differential between the two contracts is high (\$6.10 and \$12.00) with InHFixed being far more expensive than OutSFixed. This high price differential makes OutSfixed cheaper for the firm and InHFixed more profitable for the vendor.

Finally, we analyse the scenarios under high demand volatilities. At low and medium values of c_f the price differential between the two contracts is moderate (\$3.00, \$4.80 and \$5.40, \$9.00) and hence InHFixed is cheaper for the firm. At high demand volatilities even for low and medium values of c_f the firm reserves a high capacity upfront under OutSFixed (280). This makes OutSFixed more profitable for the vendor. At high values of c_f, the price differential between the two contracts is also high (\$8.20 and \$16.30) and this makes OutSFixed more attractive for the firm and InHFixed more profitable for the vendor.

Next, we present the conflict of contract choice results in Table 4. We find that except for the scenario where both c_f and demand volatility are at medium levels there is conflict in the contract choices between the firm and the vendor. Our study reveals that at low demand volatility there is more opportunity for the vendor to come up with revenue-sharing mechanisms as his loss of profit is higher compared with the firm's increase in costs if both parties pick their least preferred contracts. However, at low values of c_f and medium demand volatility, the cost increase for the firm is the maximum and here it is impossible for the vendor to make the firm move to his preferred contract. We also find that at high values of c_f when demand volatility is also high the loss in profit as well as the increase in cost for the vendor and the firm, respectively, is marginal for both the contracts. Hence, here both the parties are indifferent in their contract choice. In the other scenarios, there are opportunities for developing mechanisms that might provide win-win to both parties.

Table 4 Summary results under changing demand volatility

	Firm's full-time cost					
	Low (c_f = \$3)		*Medium (c_f = \$6)*		*High (c_f = \$12)*	
	V(%)	*F(%)*	*V(%)*	*F(%)*	*V(%)*	*F(%)*
Demand volatility						
Low	51	3	22	3	11	3
Medium	15	73			10	1
High	10	24	8	9	0.15	0.25

c_d = \$14, c_e = 0, m_d = 50%, c_v = \$1.
V: Maximum loss in profit to the vendor if he agrees to the contract preferred by firm.
F: Maximum increase in cost to the firm if he agrees to the contract preferred by vendor.
Shaded cell depicts the combination where there is incentive congruence.

6. Managerial insights and future research directions

In this paper, we examine outsourcing arrangements for back-office operations between a financial services firm and a third-party outsourcing vendor. The firm complements in-house processing with different outsourcing arrangements to meet dynamic and fluctuating demand requirements. From our discussions with an outsourcing vendor, we found that there are different kinds of contractual agreements providing varying levels of flexibility to the firm in outsourcing the workload. In particular, we consider two different kinds of contracts. We classify these contracts based on whether the firm should outsource the variable portion of the demand and keep a stable demand in-house (InHFixed), or whether a stable portion of the demand is outsourced and the variability is kept in-house (OutSFixed). The volume of workload outsourced or kept in-house can vary daily in each of those contracts.

We develop stochastic goal programming models for the firm's in-house operations in order to capture the temporal elements and the uncertainty associated with the problem. We model the contractual agreements between the firm and the outsourcing vendor as Stackelberg games where the vendor moves first with the price and the firm decides on the outsourcing volume as well as his in-house employee schedules. This paper is among the first to model in-house workforce staffing and outsourcing simultaneously in the context of 'batch' type back-office operations under stochastic and dynamic demand.

In the financial services sector there are different contractual choices made available to the management by third-party vendors. Capacity-based and volume-based contracts are two of the most widely used contracts. These contracts vary by the degree of flexibility provided in outsourcing the volatility of demand. It is imperative for the financial services provider as well as the outsourcing vendor to ascertain the economic implication of each of the contracts. We explore the contract choice problem through numerical examples to provide critical insights on each of the contracts. Demand characteristics are a key element that influences the optimal decisions of each party. Here, we analyse the effect of varying the volumes of demand under different economic parameters. We find that there is most incentive conflict between the firm and the vendor when the demand volume is low. At other parameter values, there is either incentive congruence or a possibility of congruence by sharing some of the profits.

In practice, a negotiation process between the two resolves the contract choice parties and the relative clout of each party determines which contract is chosen. Our analysis provides guidelines about the contract choice that can be used in the negotiation process. We also identify scenarios under which the vendor can come up with revenue-sharing mechanisms through which he can induce the firm to pick the more costly contract by covering the extra costs and yet make more profits.

In this paper, we have analysed two types of contracts. For future research, several other types of contracts can be investigated. For example, demand correlation over days on contract choice can be considered. Innovative contractual agreements based on information asymmetry are interesting research extensions. Revenue-sharing contracts that develop mechanisms for coordinating both the parties can be another worthwhile future research endeavour. On the basis of the contract-choice deduced in the paper, firms could also come up with contractual variations that provide improvements for one party without harming the other one. Options-type contracts that could enable pareto improvements could be a worthwhile future research extension. Finally, some of these contracts can be validated using empirical analysis. In some of these cases, modelling approaches similar to ours can be used, while others would require completely different modelling tools. Modelling the negotiation process that formally captures the contract choice mechanism is another interesting research extension.

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Appendix

Proof of Lemma 1: Since we are proving the analytical results for the single period formulation, we drop the subscript t form our notations. To prove the above result, we first represent the positive and negative deviations, S_k^+ and S_k^-, that satisfy the overtime constraint as follows:

$$S_k^+ = \begin{cases} U & \text{if } k \leqslant V^o \\ U + V^o - k & \text{if } V^o < k \leqslant U + V^o \\ 0 & \text{if } k > U + V^o \end{cases}$$

and

$$S_k^- = \begin{cases} 0 & \text{if } k < U + V^o \\ k - U - V^o & \text{if } k \geqslant U + V^o \end{cases}$$

Then, the firm's cost minimization objective can be written as:

$$\Omega^o = c_f U + p^o V^o + c_d \sum_{k=U+V^o}^{K} (k - U - V^o) p_k - c_e \sum_{k=0}^{V^o} U p_k - c_e \sum_{k=V^o+1}^{U+V^o} (U + V^o - k) p_k$$

We will show that at $p^o = c_f$, the firm incurs lower cost if it outsources zero volume and covers the demand requirements with only in-house operations rather than outsourcing V^o with U number of full-time employees.

Without any loss of generality, we pick, $U + V^o = M (< K)$.

We denote, the firm's cost when it outsources zero volume and uses M number of full-time employees as Ω_1^o and when it outsources V^o with U number of full-time employees as Ω_2^o. We need to show that $\Omega_1^o < \Omega_2^o$.

$$\Omega_1^o = c_f M + c_d \sum_{k=M}^{K} (k - M) p_k - c_e \sum_{k=0}^{K} (M - k) p_k \quad \text{(A.1)}$$

$$\begin{aligned} \Omega_2^o &= c_f U + c_f V^o + c_d \sum_{k=U+V^o}^{K} (k - U - V^o) p_k \\ &\quad - c_e \sum_{k=0}^{V^o} U p_k - c_e \sum_{k=V^o+1}^{U+V^o} (U + V^o - k) p_k \\ &= c_f M + c_d \sum_{k=M}^{K} (k - M) p_k - c_e \sum_{k=0}^{V^o} U p_k \\ &\quad - c_e \sum_{k=V^o+1}^{M} (M - k) p_k \\ &= c_f M + c_d \sum_{k=M}^{K} (k - M) p_k - c_e \sum_{k=0}^{M} (M - k) p_k \\ &\quad + c_e \sum_{k=0}^{V^o} (V^o - k) p_k \end{aligned} \quad \text{(A.2)}$$

Since, $c_e \geqslant 0$ and $V^o - k \geqslant 0$ for $0 \leqslant k \leqslant V^o$, we can conclude from (A.1) and (A.2): $\Omega_1^o < \Omega_2^o$. Therefore, at $p^o = c_f$, the firm will never outsource any volume. Hence, the optimal price charged by the vendor has to be less than c_f otherwise the vendor will earn zero revenue. When $c_e = 0$, it follows that the optimal price charged by the vendor has to be $\leqslant c_f$, that is, $p^o \leqslant c_f$. □

Proof of Lemma 2: As we had done in the proof for Proposition 1, we first represent the positive and negative deviations, S_k^+ and S_k^-, that satisfy the overtime constraint as follows:

$$S_k^+ = \begin{cases} U-k & \text{if } 0 \leqslant k \leqslant U \\ 0 & \text{if } k > U \end{cases}$$

and

$$S_k^- = \begin{cases} 0 & \text{if } 0 \leqslant k < U \\ k-U & \text{if } U \leqslant k \leqslant d_{jk}^i \\ d_{jk}^i - U & \text{if } d_{jk}^i < k \leqslant K \end{cases}$$

Then the firm's cost minimization objective can be written as follows:

$$\Omega^i = c_f U + c_d \sum_{k=U}^{d_{jk}^i} (k-U)p_k + c_d \sum_{k=d_{jk}^i+1}^{K} (d_{jk}^i - U)p_k - c_e \sum_{k=0}^{U} (U-k)p_k + p^i \sum_{k=d_{jk}^i+1}^{K} (k - d_{jk}^i)p_k$$

To prove that $p^i \geqslant c_f$ we first show that for $p^i = c_f$ the firm outsources all the workload. Let Ω_1^i be the firm's total cost if *all* the workload is outsourced and Ω_2^i be the firm's total cost if volume equal to d_{jk}^i is kept in-house with U number of full-time employees at $p^i = c_f$. We will show that $\Omega_1^i < \Omega_2^i$. d_{jk}^i and U are picked without any loss of generality.

$$\Omega_1^i = c_f \sum_{k=0}^{K} k p_k \tag{A.3}$$

Since, $c_d > c_f$, we denote $c_d = c_f + \tilde{c}$. Then

$$\begin{aligned} \Omega_2^i &= c_f U + c_d \sum_{k=U}^{d_{jk}^i} (k-U)p_k + c_d \sum_{k=d_{jk}^i+1}^{K} (d_{jk}^i - U)p_k \\ &\quad - c_e \sum_{k=0}^{U} (U-k)p_k + c_f \sum_{k=d_{jk}^i+1}^{K} (k - d_{jk}^i)p_k \\ &= c_f U \sum_{k=0}^{K} p_k + c_d \sum_{k=U}^{d_{jk}^i} (k-U)p_k + c_d \sum_{k=d_{jk}^i+1}^{K} (d_{jk}^i - U)p_k \\ &\quad - c_e \sum_{k=0}^{U} (U-k)p_k + c_f \sum_{k=d_{jk}^i+1}^{K} (k - d_{jk}^i)p_k \\ &= c_f U \sum_{k=0}^{K} p_k + (c_f + \tilde{c}) \sum_{k=U}^{d_{jk}^i} (k-U)p_k \\ &\quad + (c_f + \tilde{c}) \sum_{k=d_{jk}^i+1}^{K} (d_{jk}^i - U)p_k - c_e \sum_{k=0}^{U} (U-k)p_k \\ &\quad + c_f \sum_{k=d_{jk}^i+1}^{K} (k - d_{jk}^i)p_k \\ &= c_f U \sum_{k=0}^{U} p_k + c_f \sum_{k=0}^{K} k p_k + \tilde{c} \sum_{k=U}^{d_{jk}^i} (k-U)p_k \\ &\quad + \tilde{c} \sum_{k=d_{jk}^i+1}^{K} (d_{jk}^i - U)p_k - c_e U \sum_{k=0}^{U} p_k + c_e \sum_{k=0}^{U} k p_k \\ &= (c_f - c_e) U \sum_{k=0}^{U} p_k + c_f \sum_{k=0}^{K} k p_k + \tilde{c} \sum_{k=U}^{d_{jk}^i} (k-U)p_k \\ &\quad + \tilde{c} \sum_{k=d_{jk}^i+1}^{K} (d_{jk}^i - U)p_k + c_e \sum_{k=0}^{U} k p_k \end{aligned} \tag{A.4}$$

Clearly, $\Omega_1^i < \Omega_2^i$.

Therefore, at $p^i = c_f$, the firm incurs lower costs if it outsources all the workload. Using similar arguments as above we can easily show that for $p^i < c_f$ it is optimal for the firm to outsource the entire workload. Hence, we prove that for $p^i \leqslant c_f$, the vendor gets the entire volume of workload. For $p^i \leqslant c_f$, vendor's profit would be maximum if he charges $p^i = c_f$. So the vendor would never charge a price less than c_f. Therefore, c_f gives the lower bound for p^i. □

Proof of Lemma 3: Suppose the optimal price charged by the vendor is p^o and the optimal volume outsourced by the firm is V^o and the number of full-time employees is U. Let Δ be the difference in cost for the firm if the firm outsources an unit less than V^o and covers that by an additional full-time employee and if the firm outsources V^o with U full-time employees at outsourcing price p^o.

We will show that Δ is non-decreasing in demand. Let Ω_1^o be the cost incurred by the firm if it outsources V^o and employs U full-time employees and Ω_2^o be the cost incurred by the firm if it outsources $V^o - 1$ and employs $U+1$ full-time employees at outsourcing price p^o.

$$\Omega_1^o = c_f U + p^o V^o + c_d \sum_{k=U+V^o}^{K} (k - U - V^o)p_k - c_e \sum_{k=0}^{V^o} U p_k - c_e \sum_{k=V^o+1}^{U+V^o} (U + V^o - k)p_k \tag{A.5}$$

$$\Omega_2^o = c_f(U+1) + p^o(V^o - 1) + c_d \sum_{k=U+V^o}^{K} (k - U - V^o)p_k - c_e \sum_{k=0}^{V^o-1} (U+1)p_k - c_e \sum_{k=V^o}^{U+V^o} (U + V^o - k)p_k \tag{A.6}$$

Therefore,

$$\Delta = \Omega_2^o - \Omega_1^o = c_f - p^o - c_e \sum_{k=0}^{V^o-1} p_k \tag{A.7}$$

As demand increases $\sum_{k=0}^{V^o-1} p_k$ decreases by stochastic dominance.

Hence, Δ is non-decreasing in demand. That is if demand increases it is not cost efficient for the firm to decrease volume outsourced and increase the number of full-time employees by even an unit volume. Hence, it is not beneficial for the firm to increase the number of full-time employees and decrease volume outsourced under identical prices when demand increases. □

Proof of Theorem 2: From Lemma 3 we know that at the same outsourcing price p^o, the outsourcing volume of the firm is non-decreasing in demand. Hence, at the same price the vendor earns more revenue as demand increases. Therefore, the profit earned by the vendor at the optimal price has to be higher or at least equal to the profit earned under identical pricing. □

Proof of Theorem 3: We know from Lemma 2 that at $p^i = c_f$ the firm outsources all the volume and that $p^i \geqslant c_f$. Suppose the optimal price charged at a certain demand volume is $p^i = c_f$. Then as demand increases at p^i the firm still outsources all the volume. Therefore, the revenue and in turn the profits for the vendor increases even if he continues to charge $p^i = c_f$. However, if the optimal price charged is more then the increment in profit is also higher.

From the firm's perspective the vendor either charges $p^i = c_f$ or a price $> c_f$ as demand increases. Therefore, the firm cost has to increase because the firm now has to cover for more jobs and the outsourcing price is also non-decreasing. □

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